# Thermodynamics of Spin-1/2 Heisenberg Chains: Experiment and Theory

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## Collaborators

## Experiment

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#### Theory

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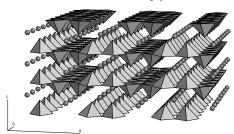
#### Introduction

We have measured the magnetic susceptibility  $\chi$  and/or specific heat  $C_p(T)$  of two insulating compounds containing spin S=1/2 chains:  $NaV_2O_5$ , and the ambient- (AP) and high-pressure (HP) forms of  $(VO)_2P_2O_7$ 

 ${\hbox{NaV}_2\hbox{O}_5}$  is structurally a two-leg spin ladder compound with one electron per V-O-V rung. According to theory, the ladder can be mapped onto a spin-1/2 chain problem.  ${\hbox{NaV}_2\hbox{O}_5}$  undergoes combined charge, lattice, and spin dimerization transitions at  $T_c=34$  K.

Goals: (1) Measure  $\chi(T)$  and  $C_p(T)$  on good single crystals of NaV<sub>2</sub>O<sub>5</sub>. (2) Compute the temperature-dependent spin gap  $\Delta$  below  $T_c$  from the  $\chi(T)$  data and compare the results with those of other measurements such as neutron scattering. (3) From the  $C_p(T)$  data, determine the relative contributions of the spin and charge/lattice degrees of freedom to the phase transition at  $T_c$ .

### Structure of NaV2O5

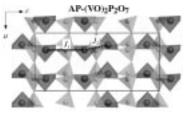


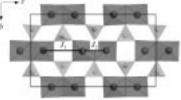
 $(VO)_2P_2O_7$ : DCJ discovered in **1987** that this S = 1/2 compound has a spin gap, and fitted his  $\chi(T)$  data by existing theoretical predictions for  $\chi(T)$  of the spin-1/2 AF alternating-exchange chain. The crystal structure suggests that the compound contains two-leg V spin ladders instead, but no predictions for  $\chi(T)$  of spin ladders existed at that time 1994: Barnes and Riera fitted DCJ's χ(T) data by a two-leg ladder model. Inelastic neutron scattering measurements on a powder by Eccleston et al. confirmed the spin ladder model 1997: Garrett et al. ruled out the spin ladder model and reconfirmed the alternating chain model from inelastic neutron scattering data on single crystals. A "troublesome" second spin gap was found, not possible for the alternating chain model, which was attributed to two-magnon bound states 1999: Yamauchi et al. deduced from NMR that (VO)2P2O7 contains two independent alternating-exchange chains, with the two spin gaps as found in the neutron measurements on single crystals

**1999:** Azuma et al. discovered a high-pressure phase of  $(VO)_2P_2O_7$  with a simpler structure containing a single type of alternating-exchange chain

Goals: (1) Measure the  $\chi(T)$  of powders and single crystals of the ambient- (AP) and high-pressure (HP) phases of  $(VO)_2P_2O_7$  and (2) test consistency with the two- and one-chain models, respectively

## Structure of AP-(VO)<sub>2</sub>P<sub>2</sub>O<sub>7</sub>



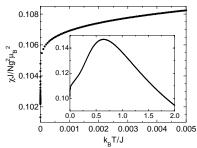


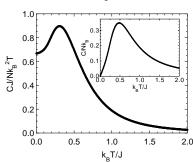
## Approach

- Carry out numerical calculations of  $\chi(T)$  for the S=1/2 uniform and alternating-exchange Heisenberg chain using Bethe ansatz calculations, quantum Monte Carlo (QMC) simulations and transfer-matrix density-matrix renormalization group (TMRG) calculations. Bethe ansatz calculations of the specific heat are also performed for the uniform chain
- · Obtain high-precision fits to the numerical data
- Model the experimental data with the theory

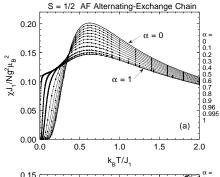
## Theory

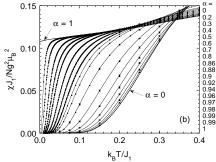
Bethe ansatz  $\chi(T)$  and specific heat C(T) data for the AF S=1/2 uniform Heisenberg chain





• The very low-temperature data are in excellent agreement with Lukyanov's 1998 exact field theory predictions Numerical QMC and TMRG data for AF alternating-exchange chains with various values of  $\alpha = J_2/J_1$ 





• The above set of solid curves is a single high-accuracy two-dimensional fit to the data with  $0 \le \alpha \le 1$  and  $0.01 \le k_B T/J_1 \le 5$ . This function has been sought by experimentalists for several decades

The  $\chi(T)$  function is recast below in terms of the average  $J=(J_1+J_2)/2$  and dimerization parameter  $\delta=(J_1-J_2)/2J$  instead of  $J_1$  and  $\alpha=J_2/J_1$ . This form is more transparent for modeling  $\chi(T)$  data for compounds containing chains with weak alternation and/or that show spin-dimerization transitions

